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# Trading Strategies for Distribution Company with Stochastic Distributed Energy Resources

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## Abstract

This paper proposes a methodology to address the trading strategies of a proactive distribution company (PDISCO) engaged in the transmission-level (TL) markets. A one-leader multi-follower bilevel model is presented to formulate the gaming framework between the PDISCO and markets. The lower-level (LL) problems include the TL day-ahead market and scenario-based real-time markets, respectively with the objectives of maximizing social welfare and minimizing operation cost. The upper-level (UL) problem is to maximize the PDISCO's profit across these markets. The PDISCO's strategic offers/bids interactively influence the outcomes of each market. Since the LL problems are linear and convex, while the UL problem is non-linear and non-convex, an equivalent primal-dual approach is used to reformulate this bilevel model to a solvable mathematical program with equilibrium constraints (MPEC). The effectiveness of the proposed model is verified by case studies.

**Keywords:** Distributed energy resources (DERs), proactive distribution company (PDISCO), electricity markets, bilevel game-theoretic model,

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multi-period AC power flow, mathematical program with equilibrium constraints (MPEC), mathematical program with primal and dual constraints (MPPDC).

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## Nomenclature

### *Sets and Indices*

	$i,j,B^{DS}$ $n,m,B^{TS}$	Index and set of distribution-level (DL) and transmission-level (TL) buses, respectively.
5	$ij,\Lambda^{DS}$ $nm,\Lambda^{TS}$	Index and set of DL feeders and TL lines, respectively.
	$l,L$ $d,D$	Index and set of DL and TL demands, respectively.
	$g,G$	Index and set of TL generators.
	$k,K$	Index and set of DL DERs.
	$t,T$	Index and set of time periods (e.g., hours per day).
10	$\omega,\Omega$	Index and set of scenarios.
	$\mathcal{M}_L, \mathcal{M}_D$	Mapping of the set of DL/TL demands onto the set of DL/TL buses respectively.
	$\mathcal{M}_G$	Mapping of the set of TL conventional generations onto the set of TL buses.
15	$\mathcal{M}_K$	Mapping of the set of DERs onto the set of DL buses.

### *Variables*

	$P_{tg}^G$	Day-ahead offer of generator $g$ at time $t$ .
	$R_{tg}^{UP}, R_{tg}^{DN}$	Day-ahead up and down regulation reserve capacities of generator $g$ at time $t$ .
20	$r_{tg\omega}^{UP}, r_{tg\omega}^{DN}$	Real-time up and down regulation power of generator $g$ at time $t$ for scenario $\omega$ .

	$\lambda_t^{DDA}$	Day-ahead offering/bidding price of the PDISCO at time $t$ .
	$P_t^{DDA}, Q_t^{DDA}$	Day-ahead offering/bidding quantity of the PDISCO at time $t$ (non-negative is offer, negative is bid).
25	$\lambda_{t\omega}^{DRT}$	Real-time offering/bidding price of the PDISCO at time $t$ for scenario $\omega$ .
	$P_{t\omega}^{DRT}, Q_{t\omega}^{DRT}$	Real-time offering/bidding quantity of the PDISCO at time $t$ for scenario $\omega$ (non-negative is offer, negative is bid).
	$P_{td\omega}^{ST}$	TL load-shedding of demand $d$ at time $t$ for scenario $\omega$ .
30	$\theta_{tn}^0, \theta_{tn\omega}$	Voltage angles of bus $n$ at day-ahead time $t$ , and at real-time time $t$ for scenario $\omega$ .
	$\lambda_{tn}^{DA}, \lambda_{tn\omega}^{RT}$	Locational marginal price (LMP) at TL bus $n$ at day-ahead time $t$ , and at real-time time $t$ for scenario $\omega$ .
35	$P_{tk}^{DER0}, Q_{tk}^{DER0}$	Active and reactive power procured from DER $k$ at day-ahead time $t$ .
	$P_{tl\omega}^{SD}, Q_{tl\omega}^{SD}$	Active and reactive power of DL load-shedding for demand $l$ at time $t$ for scenario $\omega$ .
	$Q_{ti}^{C0}, Q_{ti\omega}^C$	Reactive power from DL shunt compensator at bus $i$ at day- ahead time $t$ , and at real-time time $t$ for scenario $\omega$ .
40	$P_{t,ij}^{FD0}, Q_{t,ij}^{FD0}$	Day-ahead active and reactive power flows through DL feeder $i$ - $j$ at time $t$ .
	$P_{t,ij,\omega}^{FD}, Q_{t,ij,\omega}^{FD}$	Real-time active and reactive power flows through DL feeder $i$ - $j$ at time $t$ for scenario $\omega$ .
45	$\delta_{ti}^0, \delta_{ti\omega}$	Voltage angles of bus $i$ at day-ahead time $t$ , and at real-time time $t$ for scenario $\omega$ .
	$V_{ti}^0, V_{ti\omega}$	Voltage magnitudes of DL bus $i$ at day-ahead time $t$ , and at real-time time $t$ for scenario $\omega$ .

### Parameters

	$P_{tk\omega}^{DER}, Q_{tk\omega}^{DER}$	Active and reactive power generation realization of DER $k$ at time $t$ for scenario $\omega$ .
50	$\bar{P}_g^G, \bar{R}_g^{UP}, \bar{R}_g^{DN}$	Maximum production, maximum up and down regulation reserve capacities of generator $g$ .
	$C_g^G, C_g^{UP}, C_g^{DN}$	Day-ahead generation cost, up and down regulation reserve costs of generator $g$ .
55	$c_g^{UP}, c_g^{DN}$	Real-time up and down regulation cost of generator $g$ .
	$C_t^{DS}$	Operation cost of the PDISCO at time $t$ .
	$\lambda_{td}^{TSD}, P_{td}^{TSD}$	Day-ahead bidding price and consumption of TL demand $d$ at time $t$ .
	$\lambda_t^{DER}$	DER procurement price of the PDISCO at time $t$ .
60	$\lambda_t^{DSD}$	DL sale price at time $t$ .
	$P_{tl}^{DSD}, Q_{tl}^{DSD}$	Consumption of DL demand $l$ at time $t$ .
	$\bar{P}^{DS}$	Active power injection limit for the PDISCO.
	$\lambda_t^{ST}, \lambda_t^{SD}$	TL/DL load-shedding price at time $t$ .
	$\bar{P}_{nm}^{TS}$	Capacity limit of each TL line $nm$ .
65	$\bar{S}, \bar{S}_{ij}$	Capacity limits of the DL main substation and each DL feeder $ij$ .
	$\bar{S}_k$	Capacity limit of each DER $k$ .
	$\underline{Q}_i^C, \bar{Q}_i^C$	Reactive power limits of the DL shunt compensator at bus $i$ .
	$\underline{V}_i, \bar{V}_i$	Limits of voltage magnitude at DL bus $i$ .
70	$\tau_i$	Transformer tap ratio at DL bus $i$ .

$B_{nm}$	Susceptance of the TL line $nm$ .
$G_{ij}, B_{ij}, b_{ij}$	Conductance, susceptance and charging susceptance of the DL feeder $ij$ .

## 1. Introduction

75 Distributed energy resources (DERs) tend to occupy a high share in the distribution-level (DL) network [1, 2]. In a deregulated environment, this stimulates the distribution company (DISCO) to preferentially procure DERs' generations at low prices. In the U.S., the recent initiative named the New York Reforming Energy Vision (NY REV) [3] has addressed the regulatory changes to liberate a DL market for cost-effective use of DERs. As indicated in the NY  
80 REV, a Distributed System Platform Provider (DSPP) will modernize its distribution system to create a flexible platform for new energy products and services, to improve the overall system efficiency. Resources provided could include distributed generation (DG), energy efficiency, predictive demand management, demand response (DR), microgrids (MGs), and more. This paper is partially  
85 motivated by the NY REV and aims to establish a real-time market framework for the PDISCO procuring the DL DERs and trading in the transmission-level (TL) wholesale markets. The DL resources are selected as stochastic DERs, such as wind turbines (WTs) and photovoltaic systems (PVs), while the proactive  
90 DISCO (PDISCO) can be considered as a DSPP to play an essential role in the hierarchical trading framework. To this end, the PDISCO gets an opportunity to strategically engage in the TL markets by rationally purchasing electricity from the DL DERs. In this situation, the DISCO's trading framework becomes more complex.

95 Associated with the smart grid technology, to participate in the day-ahead and real-time markets, for each time  $t$ , the DISCO has to make a trade-off on acquiring DERs' portfolio and trading strategy (offer/bid) to maximize its profit. Crossing the two-stage markets, the transactions between the DISCO and markets are characterized in a bidirectional fashion, implying the DISCO

behaves as an active producer when providing offers, but as an active consumer when submitting bids. To highlight these features, this kind of DISCO is defined as a PDISCO in this paper. On the other hand, the PDISCO's trading strategies (offering/bidding prices and power quantities) are endogenously interrelated with the markets' outcomes (locational marginal prices (LMPs) and production/consumption quantities). Thus, the trading between the PDISCO and markets follows a typical gaming structure.

In order to capture the PDISCO's trading strategies, the PDISCO trading within markets can be formulated as a one-leader multi-follower game model, realized in a bilevel structure. Market-clearing procedures are indicated as the stage-based LL problems through DC power flow. The LL day-ahead market problem is to maximize the TL social welfare. In particular, in the real-time process, scenario-based methods [4] can be used to represent the stochastic outputs of individual DERs. Accordingly, an LL real-time market problem seeks to minimize the TL operation cost per scenario  $\omega$ . The UL problem represents the profit maximization of the PDISCO, with the strategic offers/bids constrained by AC power flow. Note that the LL problems are linear and convex, while the UL problem is non-linear and non-convex. The primal-dual approach [5] is applied to reformulate the proposed bilevel model to a solvable mathematical program with equilibrium constraints (MPEC).

Few papers are available in the technical literature to discuss the DISCO trading within the TL markets. A static bilevel model is proposed in [6] to support a DISCO's operational decision with DGs and interruptible loads (ILs) in a competitive market, while the DISCO's offers/bids are ignored by the market objective. To optimize the DISCO's day-ahead acquisition, a static bilevel model [7] is presented to purchase energy from the wholesale market, ILs and dispatchable DGs, neglecting their stochastic outputs. For the same purpose, a multi-period bilevel model is further presented in [8] to exhibit the competitiveness of ILs and the DISCO-owned DGs, but the model excludes the network constraints. To address a proposed DL market framework by interacting with the TL markets, assuming the prices from TL markets are known as parame-

ters, the PDISCO procurement strategies with the DL aggregator-based DRs [9] and stochastic DGs [10] are fully presented and discussed by one-leader multi-follower bilevel models, respectively.

At differing market stages, bilevel modeling is increasingly used to determine the trading strategies [11]. For instance, in [12], given wind power production as a set of correlated scenarios, a scenario-based bilevel model is presented to derive strategic offers for a wind producer within markets. To minimize the payment in pool markets, a multi-period bilevel model [13] is proposed to address the consumers' strategies in terms of LMPs. From procurement perspectives, to establish a robust retail market, the role of a utility is suggested as a portfolio manager [14]. For a community-scale MG, a two-stage stochastic program is used to optimize its power scheduling and bidding strategy on the basis of day-ahead prices [15].

Regarding the energy development and policy, the renewable energy procurement and the related incentive policies in the state of Illinois in the U.S. are reviewed and discussed in [16]. The economic value of wind-based DGs is addressed in [17] by applying a Longstaff–Schwartz option pricing method. To evaluate the impact of residential DR choices considering uncertainties, a probabilistic methodology is presented to trade DR in a distribution energy market [18]. In competitive energy markets, the benefits of the Community Energy Storage (CES) devices are analyzed in [19] for optimizing the power system operations. Storage-like devices are also investigated in load leveling [20] through complementarity constraints and a new and exact relaxation method. In [21], new functionalities of utilities and distributed electricity prosumers are identified and cleared with the retail electricity market price. In order to facilitate large-scale EV charging facilities and energy storage devices participating in frequency regulation, [22] proposes control strategies to enhance the integration of renewable energy.

In view of the context above, the contributions of this paper are threefold:

- 1) Present a framework for the PDISCO trading in the day-ahead and real-time markets.



2) Propose a bilevel model for the PDISCO to derive continuous strategic offers and bids by rationally procuring the DERs' portfolio.

3) Reformulate the PDISCO's game-theoretic model to an MPEC by replacing the LL market problems with an equivalent primal-dual approach.

The rest of this paper is organized as follows. Section 2 presents the framework of the PDISCO trading in TL markets and procuring from DL DERs. The corresponding bilevel game-theoretic model is proposed in Section 3, in which the primal-dual approach is also indicated in detail. In Section 4, the numerical results of the case studies are reported and analyzed. Finally, some relevant remarks are concluded in Section 5.

## 2. PDISCO Trading in Markets

Till now, at the transmission level, electricity markets are normally recognized as wholesale markets and realized as pools. For short-term transactions, the electricity can be traded in the day-ahead and real-time markets. Each wholesale market follows a auction framework, which is commonly applied to find a trade-off between the offers and bids submitted by producers and consumers, respectively. Each offer/bid is specified as a set of price-quantity pairs. On the other hand, if the transmission network is considered in a market-clearing procedure, a LMP occurs at each bus, covering line congestion and line losses [5].

As shown in Fig.1, to trade in the TL markets, the PDISCO has two optional strategies to take part in each market, i.e., offer or bid in the day-ahead market and real-time market per scenario  $\omega$ . For two-stage market operation, the day-ahead market is cleared prior to the real-time markets. Therefore, at each time  $t$ , the PDISCO's strategic day-ahead offer/bid  $(\lambda_t^{DDA}, P_t^{DDA})$  goes with the day-ahead market aiming to meet the TL demands by generators' offers, while each real-time market balances the PDISCO's strategic real-time offer/bid  $(\lambda_{t\omega}^{DRT}, P_{t\omega}^{DRT})$  with the TL regulation power and/or load-shedding.

Crossing the markets, to maximize profit, the PDISCO's power quantity

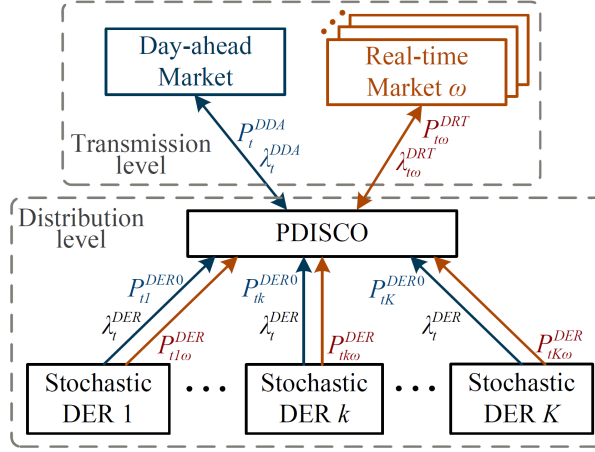


Figure 1: PDISCO trading in markets.

$P_t^{DDA}$  is scheduled by rationally procuring the DERs' portfolios (a set of  $P_{tk}^{DER0}$ ) against physical DL constraints at the day-ahead stage, and the scenario-based power quantity  $P_{t\omega}^{DRT}$  is determined as the variation pertaining to the total DERs' outputs ( $\sum_k P_{tk\omega}^{DER}$ ) versus network constraints at the real-time stage. At each time  $t$ : If  $P_t^{DDA}/P_{t\omega}^{DRT}$  is non-negative, the PDISCO behaves as an active producer with a lower offering price  $\lambda_t^{DDA}/\lambda_{t\omega}^{DRT}$ ; If this quantity is negative, the PDISCO acts as an active consumer with a higher bidding price. For the stochastic DERs, the PDISCO's procurements are settled with the price  $\lambda_t^{DER}$ , and the scenario-based method [4] can be used to represent the individual DERs' uncertainties.

### 3. Problem Formulation

Participating in the TL markets, the PDISCO's strategic trading problem can be formulated as a bilevel one-leader multi-follower game-theoretic model [5]. One follower is characterized as an LL day-ahead market problem with the goal to maximize the TL social welfare. Other followers are scenario-based LL real-time market problems. An LL problem per scenario  $\omega$  seeks to achieve the cost minimization during the real-time TL balancing. In particular, to

obtain the trade-offs between the markets, the offering/bidding strategies of the PDISCO are conducted by the UL problem with the purpose of minus-profit minimization.

### 3.1. Assumptions

The assumptions of the proposed bilevel model are as follows:

- 1) The day-ahead and real-time offering/bidding strategies of a single PDISCO are considered to be involved in the corresponding TL markets.
- 2) The DL network is assumed to be operated and owned by the PDISCO, and interconnected to the TL network via only one main substation.
- 3) Only the active power is considered to be traded in markets. At each time  $t$ , we assume the reactive power exchanging between DL and TL can be balanced at the DL main substation.
- 4) The PDISCO can explicitly anticipate the impacts of its strategic offers/bids, versus the markets' outcomes. One offer/bid per time  $t$  for each market.

### 3.2. LL problems

Note that the outcomes of the LL problems can directly affect the PDISCO's offering/bidding strategies in the UL problem. Thus, the corresponding LL problem formulations are given below.

- 1) LL day-ahead market problem:

$$\begin{aligned}
& \underset{\Xi^{DAT}}{\text{Minimize}} \sum_{t,g} (C_g^G P_{tg}^G + C_g^{UP} R_{tg}^{UP} + C_g^{DN} R_{tg}^{DN}) \\
& + \sum_t \lambda_t^{DDA} P_t^{DDA} - \sum_{t,d} \lambda_{td}^{TSD} P_{td}^{TSD} \\
& \text{s.t.}
\end{aligned} \tag{1.1}$$

For PDISCO's location bus  $I$ :

$$\begin{aligned}
& \sum_{(g:I) \in \mathcal{M}_G} P_{tg}^G + P_t^{DDA} = \sum_{Im \in \Lambda^{TS}} B_{Im} (\theta_{tI}^0 - \theta_{tm}^0) : \lambda_{tI}^{DA} \\
& , \forall t
\end{aligned} \tag{1.2}$$

$$-\bar{P}^{DS} \leq P_t^{DDA} \leq \bar{P}^{DS} : \beta_t^{0-}, \beta_t^{0+}, \forall t \quad (1.3)$$

For other buses:

$$\sum_{(g:n) \in \mathcal{M}_G} P_{tg}^G - P_{td}^{TSD} = \sum_{nm \in \Lambda^{TS}} B_{nm} (\theta_{tn}^0 - \theta_{tm}^0) : \lambda_{tn}^{DA}, \forall t, n_{(d:n) \in \mathcal{M}_D} \quad (1.4)$$

$$0 \leq P_{tg}^G + R_{tg}^{UP} \leq \bar{P}_{tg}^G : \gamma_{tg}^{0-}, \gamma_{tg}^{0+}, \forall t, g \quad (1.5)$$

$$0 \leq P_{tg}^G - R_{tg}^{DN} : \eta_{tg}^0, \forall t, g \quad (1.6)$$

$$0 \leq R_{tg}^{UP} \leq \bar{R}_g^{UP} : \phi_{tg}^{0-}, \phi_{tg}^{0+}, \forall t, g \quad (1.7)$$

$$0 \leq R_{tg}^{DN} \leq \bar{R}_g^{DN} : \psi_{tg}^{0-}, \psi_{tg}^{0+}, \forall t, g \quad (1.8)$$

$$-\bar{P}_{nm}^{TS} \leq B_{nm} (\theta_{tn}^0 - \theta_{tm}^0) \leq \bar{P}_{nm}^{TS} : \mu_{t,nm}^{0-}, \mu_{t,nm}^{0+}, \forall t, nm \in \Lambda^{TS} \quad (1.9)$$

$$-\pi \leq \theta_{tn}^0 \leq \pi : \sigma_{tn}^{0-}, \sigma_{tn}^{0+}, \forall t, n \quad (1.10)$$

$$\theta_{t1}^0 = 0 : \zeta_t^0, \forall t, n = 1 \quad (1.11)$$

where  $\Xi^{DAT} = \{P_{tg}^G, R_{tg}^{UP}, R_{tg}^{DN}, P_t^{DDA}, \theta_{tn}^0\}$  is the variable set of the LL day-ahead problem.

The day-ahead market clearing is modeled within the LL problem (1). The  
 230 objective function (1.1) is to minimize the minus (maximize) TL social welfare, i.e., the generation and reserve cost of individual generators plus the PDISCO's strategic offers/bids (non-negative  $P_t^{DDA}$  is offer, while the negative is bid,  $\forall t$ ), and minus the revenue of sales to the other TL demands. We assume all the generators are dispatchable. DC power flow is adopted to formulate the TL  
 235 operation conditions. At each time  $t$ : Constraints (1.2) and (1.4) represent the power balance at the PDISCO's location bus  $I$  and the other TL buses, respectively. Constraints (1.3) impose the bounds of the PDISCO's offering/bidding quantity  $P_t^{DDA}$ . Constraints (1.5) (1.6) (1.7) and (1.8) specify the capacity limits of the up/down reserve  $R_{tg}^{UP}/R_{tg}^{DN}$  for generator  $g$ . Constraints (1.9) indicate  
 240 the capacity limits of the TL line  $n-m$ . Constraints (1.10) express the range of the TL voltage angles, and constraints (1.11) set TL bus 1 as the reference bus. Correspondingly, the dual variables for each group of constraints are indicated

at the right side of a colon.

2) LL real-time market problems:

$$\begin{aligned} & \left\{ \begin{aligned} & \text{Minimize}_{\Xi^{RTT}} \sum_{t,g} (c_g^{UP} r_{tg\omega}^{UP} - c_g^{DN} r_{tg\omega}^{DN}) \\ & + \sum_t \lambda_{t\omega}^{DRT} P_{t\omega}^{DRT} + \sum_{t,d} \lambda_t^{ST} P_{td\omega}^{ST} \end{aligned} \right. \quad (2.1) \end{aligned}$$

s.t.

For PDISCO's location bus  $I$ :

$$\begin{aligned} & \sum_{(g:I) \in \mathcal{M}_G} (r_{tg\omega}^{UP} - r_{tg\omega}^{DN}) + P_{t\omega}^{DRT} = \sum_{Im \in \Lambda^{TS}} B_{Im} \\ & (\theta_{tI\omega} - \theta_{tI}^0 - \theta_{tm\omega} + \theta_{tm}^0) : \lambda_{tI\omega}^{RT}, \forall t \end{aligned} \quad (2.2)$$

$$-\bar{P}^{DS} \leq P_{t\omega}^{DRT} \leq \bar{P}^{DS} : \beta_{t\omega}^-, \beta_{t\omega}^+, \forall t \quad (2.3)$$

For other buses:

$$\begin{aligned} & \sum_{(g:I) \in \mathcal{M}_G} (r_{tg\omega}^{UP} - r_{tg\omega}^{DN}) + P_{td\omega}^{ST} = \sum_{nm \in \Lambda^{TS}} B_{nm} \\ & (\theta_{tn\omega} - \theta_{tn}^0 - \theta_{tm\omega} + \theta_{tm}^0) : \lambda_{tn\omega}^{RT}, \forall t, n_{(d:n) \in \mathcal{M}_D} \end{aligned} \quad (2.4)$$

$$0 \leq r_{tg\omega}^{UP} \leq R_{tg}^{UP} : \phi_{tg\omega}^-, \phi_{tg\omega}^+, \forall t, g \quad (2.5)$$

$$0 \leq r_{tg\omega}^{DN} \leq R_{tg}^{DN} : \psi_{tg\omega}^-, \psi_{tg\omega}^+, \forall t, g \quad (2.6)$$

$$0 \leq P_{td\omega}^{ST} \leq P_{td}^{TSD} : \nu_{td\omega}^-, \nu_{td\omega}^+, \forall t, d \quad (2.7)$$

$$\begin{aligned} & -\bar{P}_{nm}^{TS} \leq B_{nm} (\theta_{tn\omega} - \theta_{tm\omega}) \leq \bar{P}_{nm}^{TS} : \mu_{t,nm,\omega}^-, \\ & \mu_{t,nm,\omega}^+, \forall t, nm \in \Lambda^{TS} \end{aligned} \quad (2.8)$$

$$-\pi \leq \theta_{tn\omega} \leq \pi : \sigma_{tn\omega}^-, \sigma_{tn\omega}^+, \forall t, n \quad (2.9)$$

$$\theta_{t1\omega} = 0 : \zeta_{t\omega}, \forall t, n = 1 \left. \vphantom{\theta_{t1\omega}} \right\} \forall \omega. \quad (2.10)$$

where  $\Xi^{RTT} = \{r_{tg\omega}^{UP}, r_{tg\omega}^{DN}, P_{t\omega}^{DRT}, P_{td\omega}^{ST}, \theta_{tn\omega}\}$  is the variable set of each LL real-time problem per scenario  $\omega$ . At this stage, the day-ahead market variables  $R_{tg}^{UP}$ ,  $R_{tg}^{DN}$  and  $\theta_{tn}^0$  are recognized as parameters.

The real-time market is cleared by (2). For each scenario  $\omega$ , the objective (2.1) indicates the TL operation cost minimization, which comprises the regula-

tion cost of each generator  $g$ , the strategic offers/bids of the PDISCO, and the  
 250 cost of load-shedding invocations. On the basis of DC power flow, at each time  
 $t$  per scenario  $\omega$ : Constraints (2.2) guarantee the power balance at the PDISCO  
 bus  $I$  to deal with its strategic offering/bidding quantity  $P_{t\omega}^{DRT}$ , which is further  
 limited in (2.3). For other buses, constraints (2.4) ensure the power balance at  
 each bus when involuntary load-shedding occurs. Constraints (2.5) and (2.6)  
 255 enforce the up and down regulation  $(r_{tg\omega}^{UP}, r_{tg\omega}^{DN})$  of each generator should not  
 exceed its up and down reserve capacity  $(R_{tg}^{UP}, R_{tg}^{DN})$ , respectively. Constraints  
 (2.7) indicate the TL load-shedding limits. Constraints (2.8) identify the trans-  
 mission capacity bounds. Constraints (2.9) and (2.10) limit the real-time TL  
 voltage angle at bus  $n$ . The related dual variables are also separated by a colon.

### 260 3.3. UL problem

Interrelated with the LL problems, the offers/bids determined by the UL  
 problem inevitably influence the markets' outcomes in the LL problems. There-  
 fore, the UL problem covers the PDISCO's offering/bidding strategies in indi-  
 vidual markets, DL network constraints, and markets' arguments regarding the  
 LMPs and production/consumption quantities.

$$\begin{aligned}
 & \underset{\substack{\Xi^{DAT} \cup \Xi^{RTT} \\ \cup \Xi^{PDISCO} \cup \Xi^{Dual}}}{\text{Minimize}} & - \sum_t (\lambda_{tI}^{DA} - C_t^{DS}) P_t^{DDA} \\
 & + \sum_{t,k} \lambda_t^{DER} P_{tk}^{DER0} - \sum_{t,l} \lambda_t^{DSD} P_{tl}^{DSD} + \\
 & \mathbb{E} \left[ - \sum_t (\lambda_{tI\omega}^{RT} - C_t^{DS}) P_{t\omega}^{DRT} + \sum_{t,l} (\lambda_t^{SD} + \lambda_t^{DSD}) P_{tl\omega}^{SD} \right. \\
 & \left. - \sum_{t,k} (\lambda_{tI\omega}^{RT} - \lambda_t^{DER}) (P_{tk\omega}^{DER} - P_{tk}^{DER0}) \right] \tag{3.1}
 \end{aligned}$$

s.t.

PDISCO day-ahead constraints (3.2)-(3.16):

$$P_t^{DDA} \leq \sum_k P_{tk}^{DER0} - \sum_l P_{tl}^{DSD}, \forall t \tag{3.2}$$

For the main substation (PDISCO's reference bus 1):

$$-P_t^{DDA} + \sum_{(k:1) \in \mathcal{M}_K} P_{tk}^{DER0} - P_{t1}^{DSD} = \sum_{1j \in \Lambda^{DS}} P_{t,1j}^{FD0}, \forall t \quad (3.3)$$

$$\begin{aligned} & -Q_t^{DDA} + \sum_{(k:1) \in \mathcal{M}_K} Q_{tk}^{DER0} - Q_{t1}^{DSD} + Q_{t1}^{C0} \\ & = \sum_{1j \in \Lambda^{DS}} Q_{t,1j}^{FD0}, \forall t \end{aligned} \quad (3.4)$$

$$V_{t1}^0 = 1, \forall t \quad (3.5)$$

$$\delta_{t1}^0 = 0, \forall t \quad (3.6)$$

$$(P_t^{DDA})^2 + (Q_t^{DDA})^2 \leq \bar{S}, \forall t \quad (3.7)$$

For the other buses:

$$\sum_{(k:i) \in \mathcal{M}_K} P_{tk}^{DER0} - P_{ti}^{DSD} = \sum_{ij \in \Lambda^{DS}} P_{t,ij}^{FD0}, \forall t, i_{(l:i) \in \mathcal{M}_L} \quad (3.8)$$

$$\begin{aligned} & \sum_{(k:i) \in \mathcal{M}_K} Q_{tk}^{DER0} - Q_{ti}^{DSD} + Q_{ti}^{C0} = \sum_{ij \in \Lambda^{DS}} Q_{t,ij}^{FD0}, \\ & \forall t, i_{(l:i) \in \mathcal{M}_L} \end{aligned} \quad (3.9)$$

$$\begin{aligned} P_{t,ij}^{FD0} &= -\tau_i (V_{ti}^0)^2 G_{ij} + V_{ti}^0 V_{tj}^0 [G_{ij} \cos(\delta_{ti}^0 - \delta_{tj}^0) \\ &+ B_{ij} \sin(\delta_{ti}^0 - \delta_{tj}^0)], \forall t, ij \in \Lambda^{DS} \end{aligned} \quad (3.10)$$

$$\begin{aligned} Q_{t,ij}^{FD0} &= \tau_i (V_{ti}^0)^2 B_{ij} - 0.5b_{ij} + V_{ti}^0 V_{tj}^0 [G_{ij} \sin(\delta_{ti}^0 - \delta_{tj}^0) \\ &- B_{ij} \cos(\delta_{ti}^0 - \delta_{tj}^0)], \forall t, ij \in \Lambda^{DS} \end{aligned} \quad (3.11)$$

$$(P_{t,ij}^{FD0})^2 + (Q_{t,ij}^{FD0})^2 \leq (\bar{S}_{ij})^2, \forall t, ij \in \Lambda^{DS} \quad (3.12)$$

$$(P_{tk}^{DER0})^2 + (Q_{tk}^{DER0})^2 \leq (\bar{S}_k)^2, \forall t, k \quad (3.13)$$

$$\underline{Q}_i^C \leq Q_{ti}^{C0} \leq \bar{Q}_i^C, \forall t, i \quad (3.14)$$

$$\underline{V}_i \leq V_{ti}^0 \leq \bar{V}_i, \forall t, i \quad (3.15)$$

$$-\pi \leq \delta_{ti}^0 \leq \pi, \forall t, i \quad (3.16)$$

PDISCO real-time constraints (3.17)-(3.33):

$$P_{t\omega}^{DRT} \leq \sum_k (P_{tk\omega}^{DER} - P_{tk}^{DER0}) + \sum_l P_{tl\omega}^{SD}, \forall t, \omega \quad (3.17)$$

For the main substation (PDISCO's reference bus 1):

$$\begin{aligned}
& -P_{t\omega}^{DRT} + \sum_{(k:1) \in \mathcal{M}_K} (P_{tk\omega}^{DER} - P_{tk}^{DER0}) + P_{t1\omega}^{SD} \\
& = \sum_{1j \in \Lambda^{DS}} (P_{t,1j\omega}^{FD} - P_{t,1j}^{FD0}), \forall t, \omega
\end{aligned} \tag{3.18}$$

$$\begin{aligned}
& -Q_{t\omega}^{DRT} + \sum_{(k:1) \in \mathcal{M}_K} (Q_{tk\omega}^{DER} - Q_{tk}^{DER0}) + Q_{t1\omega}^{SD} + Q_{t1\omega}^C \\
& - Q_{t1}^{C0} = \sum_{1j \in \Lambda^{DS}} (Q_{t,1j\omega}^{FD} - Q_{t,1j}^{FD0}), \forall t, \omega
\end{aligned} \tag{3.19}$$

$$V_{t1\omega} = 1, \forall t, \omega \tag{3.20}$$

$$\delta_{t1\omega} = 0, \forall t, \omega \tag{3.21}$$

$$(P_t^{DDA} + P_{t\omega}^{DRT})^2 + (Q_t^{DDA} + Q_{t\omega}^{DRT})^2 \leq \bar{S}^2, \forall t, \omega \tag{3.22}$$

For the other buses:

$$\begin{aligned}
& \sum_{(k:i) \in \mathcal{M}_K} (P_{tk\omega}^{DER} - P_{tk}^{DER0}) + P_{tl\omega}^{SD} \\
& = \sum_{ij \in \Lambda^{DS}} (P_{t,ij,\omega}^{FD} - P_{t,ij}^{FD0}), \forall t, i_{(l:i) \in \mathcal{M}_L}, \omega
\end{aligned} \tag{3.23}$$

$$\begin{aligned}
& \sum_{(k:i) \in \mathcal{M}_K} (Q_{tk\omega}^{DER} - Q_{tk}^{DER0}) + Q_{tl\omega}^{SD} + Q_{ti\omega}^C - Q_{ti}^{C0} \\
& = \sum_{ij \in \Lambda^{DS}} (Q_{t,ij,\omega}^{FD} - Q_{t,ij}^{FD0}), \forall t, i_{(l:i) \in \mathcal{M}_L}, \omega
\end{aligned} \tag{3.24}$$

$$\begin{aligned}
P_{t,ij,\omega}^{FD} & = -\tau_i V_{ti\omega}^2 G_{ij} + V_{ti\omega} V_{tj\omega} [G_{ij} \cos(\delta_{ti\omega} - \delta_{tj\omega}) \\
& + B_{ij} \sin(\delta_{ti\omega} - \delta_{tj\omega})], \forall t, ij \in \Lambda^{DS}, \omega
\end{aligned} \tag{3.25}$$

$$\begin{aligned}
Q_{t,ij,\omega}^{FD} & = \tau_i V_{ti\omega}^2 B_{ij} - 0.5 b_{ij} + V_{ti\omega} V_{tj\omega} [G_{ij} \sin(\delta_{ti\omega} - \delta_{tj\omega}) \\
& - B_{ij} \cos(\delta_{ti\omega} - \delta_{tj\omega})], \forall t, ij \in \Lambda^{DS}, \omega
\end{aligned} \tag{3.26}$$

$$(P_{t,ij,\omega}^{FD})^2 + (Q_{t,ij,\omega}^{FD})^2 \leq (\bar{S}_{ij})^2, \forall t, ij \in \Lambda^{DS}, \omega \tag{3.27}$$

$$0 \leq P_{tl\omega}^{SD} \leq P_{tl}^{DSD}, \forall t, l, \omega \tag{3.28}$$

$$0 \leq Q_{tl\omega}^{SD} \leq Q_{tl}^{DSD}, \forall t, l, \omega \tag{3.29}$$

$$P_{tl}^{DSD} Q_{tl\omega}^{SD} - P_{tl\omega}^{SD} Q_{tl}^{DSD} = 0, \forall t, l, \omega \tag{3.30}$$

$$\underline{Q}_i^C \leq Q_{ti\omega}^C \leq \bar{Q}_i^C, \forall t, i, \omega \tag{3.31}$$

$$\underline{V}_i \leq V_{ti\omega} \leq \bar{V}_i, \forall t, i, \omega \tag{3.32}$$



$$-\pi \leq \delta_{ti\omega} \leq \pi, \forall t, i, \omega \quad (3.33)$$

$$\lambda_{tI}^{DA}, P_t^{DDA} \in \arg(1)^{DA} \quad (3.34)$$

$$\lambda_{tI\omega}^{RT}, P_{t\omega}^{DRT} \in \arg(2)^{RT}_\omega \quad (3.35)$$

where  $\Xi^{PDISCO} = \{\lambda_t^{DDA}, \lambda_{t\omega}^{DRT}, P_{tk}^{DER0}, Q_{tk}^{DER0}, P_{tl\omega}^{SD}, Q_{tl\omega}^{SD}, Q_{ti}^{C0}, P_{t,ij}^{FD0}, Q_{t,ij}^{FD0}, \delta_{ti}^0, V_{ti}^0, Q_{ti\omega}^C, P_{t,ij,\omega}^{FD}, Q_{t,ij,\omega}^{FD}, \delta_{ti\omega}, V_{ti\omega}\}$  is the variable set of the UL PDISCO problem (3).  $\Xi^{DUAL}$  is the set of the dual variables.

To minimize the PDISCO's minus-profit (maximize profit), the objective  
 265 (3.1) of the UL problem is made up of day-ahead and real-time aspects. The day-ahead aspect include the revenue/cost from the PDISCO's strategic offers/bids in the day-ahead market, the procurements from the individual DERs, and the revenue of electricity sales to the DL demands. The real-time aspect is the expected minus-profit according to the revenue/cost from the PDISCO's  
 270 strategic offers/bids in the real-time markets, the penalty of possible DL load-shedding, and the revenue/cost from the DERs' production deviations. The PDISCO's main substation located at the TL bus  $I$  is seen as the DL reference bus 1. AC power flow is used to formulate the PDISCO's operation model.

For the day-ahead aspect, at each time  $t$ : Constraints (3.2) illustrate the upper  
 275 bound of the PDISCO's day-ahead offering/bidding quantity. Constraints (3.3) and (3.4) enforce the AC power balance at the DL reference bus, which keeps the voltage magnitude and voltage angle as constant values with constraints (3.5) and (3.6). Constraints (3.7) impose the capacity limits of the main substation. Constraints (3.8) and (3.9) represent the AC power balance  
 280 at the other buses. Constraints (3.10) and (3.11) depict the AC power flow through feeder  $i$ - $j$ , which is restricted by constraints (3.12). Constraints (3.13) specify the production limits of DER  $k$ . The capacity of each compensator is bounded in constraints (3.14). Constraints (3.15) and (3.16) limit the voltage magnitude and angle at each DL bus.

285 For the real-time aspect, at each time  $t$  per scenario  $\omega$ : Constraints (3.17) indicate the upper bound of the PDISCO's real-time offering/bidding quantity. Constraints (3.18) and (3.19) guarantee the AC power balance at the DL

reference bus. Constraints (3.20), (3.21), (3.32) and (3.33) enforce the value of voltage magnitude and angle at each bus  $n$ . The capacity limits of the main sub-station, each feeder and each compensator are constrained by (3.22), (3.27) and (3.31), respectively. Constraints (3.23) and (3.24) realize the AC power balance for the other DL buses. The AC power flow is expressed in constraints (3.25) and (3.26). Constraints (3.28)-(3.30) keep the demand power factor constant if DL load-shedding occurs. Note that the PDISCO's offering/bidding prices  $\lambda_t^{DDA}$  and  $\lambda_{t\omega}^{DRT}$  are UL decision variables treated as parameters in the LL problems. Thus, constraints (3.24) and (3.35) represent the interactive impacts between the PDISCO's trading strategies (offering/bidding price and quantity) and markets' outcomes.

### 3.4. MPEC

Note that the proposed bilevel model is put forward by the non-linear UL problem and the linear LL problems. Thus, this bilevel model can be translated into a single-level model by replacing the LL markets' problems with their first-order optimality conditions, and this renders an MPEC. Since the primal-dual approach is more tractable and efficient than the Karush-Kuhn-Tucker (KKT) conditions for off-the-shell branch-and-cut software [5], the former approach is applied in this paper.

The primal-dual approach renders a mathematical program with primal and dual constraints (MPPDC), which is carried out on the transformation of each LL problem. The MPPDCs for the day-ahead market problem and real-time market problems are indicated in (4) and (5), respectively. Constraints (4.1)-(4.5) are the dual constraints of the primal constraints (1.2)-(1.11). Constraint (4.6) is the associated strong duality equality of problem (1). For each scenario  $\omega$ , constraints (5.1)-(5.5) are the dual constraints of the primal constraints (2.2)-(2.10), and constraint (5.6) is the associated strong duality equality of problem (2). The strong duality constraint ensures the equality of the primal and dual objective values, one per LL market problem.

For the LL day-ahead market problem:

$$C_g^G + \lambda_{tn}^{DA} + \gamma_{tg}^{0+} - \gamma_{tg}^{0-} - \eta_{tg}^0 = 0, \forall t, g(g:n) \in \mathcal{M}_G \quad (4.1)$$

$$C_g^{UP} + \gamma_{tg}^{0+} - \gamma_{tg}^{0-} + \phi_{tg}^{0+} - \phi_{tg}^{0-} = 0, \forall t, g \quad (4.2)$$

$$C_g^{DN} + \eta_{tg}^0 + \psi_{tg}^{0+} - \psi_{tg}^{0-} = 0, \forall t, g \quad (4.3)$$

$$\lambda_t^{DDA} + (\lambda_{tI}^{DA})_{n=I} + \beta_t^{0+} - \beta_t^{0-} = 0, \forall t \quad (4.4)$$

$$\begin{aligned} & \sum_{nm \in \Lambda^{TS}} B_{nm} (\lambda_{tn}^{DA} - \lambda_{tm}^{DA}) + \sigma_{tn}^{0+} - \sigma_{tn}^{0-} \\ & + \sum_{nm \in \Lambda^{TS}} B_{nm} (\mu_{t,nm}^{0+} - \mu_{t,mn}^{0+}) + (\zeta_t^0)_{n=1} \\ & - \sum_{nm \in \Lambda^{TS}} B_{nm} (\mu_{t,nm}^{0-} - \mu_{t,mn}^{0-}) = 0, \forall t, n \end{aligned} \quad (4.5)$$

$$\begin{aligned} & \sum_{t,g} (C_g^G P_{tg}^G + C_g^{UP} R_{tg}^{UP} + C_g^{DN} R_{tg}^{DN}) + \sum_t \lambda_t^{DDA} P_t^{DDA} \\ & + \sum_t \bar{P}^{DS} (\beta_t^{0+} + \beta_t^{0-}) + \sum_{t,g} \gamma_{tg}^{0+} \bar{P}_{tg}^G + \sum_{t,g} \phi_{tg}^{0+} \bar{R}_g^{UP} \\ & + \sum_{t,nm \in \Lambda^{TS}} \bar{P}_{nm}^{TS} (\mu_{t,nm}^{0+} + \mu_{t,nm}^{0-}) + \sum_{t,n} \lambda_{tn}^{DA} P_{t(d:n) \in \mathcal{M}_D}^{TSD} \\ & + \sum_{t,g} \psi_{tg}^{0+} \bar{R}_g^{DN} + \sum_{t,n} \pi (\sigma_{tn}^{0+} + \sigma_{tn}^{0-}) = 0 \end{aligned} \quad (4.6)$$

For the LL real-time market problems:

$$c_g^{UP} + \lambda_{tn\omega}^{RT} + \phi_{tg\omega}^+ - \phi_{tg\omega}^- = 0, \forall t, g(g:n) \in \mathcal{M}_G, \omega \quad (5.1)$$

$$c_g^{DN} - \lambda_{tn\omega}^{RT} + \psi_{tg\omega}^+ - \psi_{tg\omega}^- = 0, \forall t, g(g:n) \in \mathcal{M}_G, \omega \quad (5.2)$$

$$\lambda_{t\omega}^{DRT} + (\lambda_{tI\omega}^{RT})_{n=I} + \beta_{t\omega}^+ - \beta_{t\omega}^- = 0, \forall t, \omega \quad (5.3)$$

$$\lambda_t^{ST} + \lambda_{t(n:d) \in \mathcal{M}_D \omega}^{RT} + \nu_{td\omega}^+ - \nu_{td\omega}^- = 0, \forall t, d, \omega \quad (5.4)$$

$$\begin{aligned} & \sum_{nm \in \Lambda^{TS}} B_{nm} (\lambda_{tn\omega}^{RT} - \lambda_{tm\omega}^{RT}) + \sigma_{tn\omega}^+ - \sigma_{tn\omega}^- \\ & + \sum_{nm \in \Lambda^{TS}} B_{nm} (\mu_{t,nm,\omega}^+ - \mu_{t,mn,\omega}^+) + (\zeta_{t\omega})_{n=1} \\ & - \sum_{nm \in \Lambda^{TS}} B_{nm} (\mu_{t,nm,\omega}^- - \mu_{t,mn,\omega}^-) = 0, \forall t, n, \omega \end{aligned} \quad (5.5)$$

$$\sum_{t,g} (c_g^{UP} r_{tg\omega}^{UP} + c_g^{DN} r_{tg\omega}^{DN}) + \sum_t \lambda_{t\omega}^{DRT} P_{t\omega}^{DRT}$$

$$\begin{aligned}
& + \sum_t \bar{P}^{DS} (\beta_{t\omega}^+ + \beta_{t\omega}^-) + \sum_{t,d} \lambda_t^{ST} P_{td\omega}^{ST} \\
& + \sum_{t,g} \phi_{tg\omega}^+ R_{tg}^{UP} + \sum_{t,g} \psi_{tg\omega}^+ R_{tg}^{DN} + \sum_{t,n} \pi (\sigma_{tn\omega}^+ + \sigma_{tn\omega}^-) \\
& + \sum_{t,d} \nu_{td\omega}^+ P_{td}^{TSD} - \sum_{t,n} \lambda_{n\omega}^{RT} \sum_{nm \in \Lambda^{TS}} B_{nm} (\theta_{tn}^0 - \theta_{tm}^0) \\
& + \sum_{nm \in \Lambda^{TS}} \bar{P}_{t,nm}^{TS} (\mu_{t,nm,\omega}^+ + \mu_{t,nm,\omega}^-) = 0, \forall \omega
\end{aligned} \tag{5.6}$$

Substituting each LL market problem with the corresponding MPPDC, the proposed bilevel model is transformed into a single-level optimization problem, as expressed in (6). Thus, the commercial off-the-shelf large-scale non-linear optimization solver CONOPT3 [23] can be employed to solve this reformulated non-linear model.

$$\begin{aligned}
& \underset{\substack{\Xi^{DAT} \cup \Xi^{RTT} \\ \cup \Xi^{PDISCO} \cup \Xi^{Dual}}}{\text{Minimize}} \quad (3.1) \\
& \text{s.t.}
\end{aligned} \tag{6}$$

PDISCO's problem constraints:

$$(3.2) - (3.35);$$

Day-ahead market problem MPPDC constraints:

$$(1.2) - (1.11) \text{ and } (4);$$

Real-time market problems MPPDC constraints:

$$(2.2) - (2.10) \text{ and } (5).$$

#### 4. Case Studies

In this section, a modified 14-bus DL network [24] interconnected to a 9-bus TL network [25] is used to validate the effectiveness of the proposed bilevel model for deriving the PDISCO's strategic offers/bids and markets' performances.

To simulate the uncertainties of individual DERs, the scenario handling approach [4] is adopted to create 1000 scenarios and reduce to 15 scenarios for the case studies.

All cases are implemented on a 3.6 GHz Intel Core i7 processor based on a  
 330 64-bit Windows 7 system carrying 16 GB of RAM, and solved by CONOPT3  
 with GAMS 24.4.1 [23].

#### 4.1. Data

For the PDISCO-owned 14-bus network: The capacities of the main sub-  
 station  $\bar{S}$  and each feeder  $\bar{S}_{ij}$  are set to 200 MVA and 100 MVA, respectively.  
 335 The voltage keeps 1 p.u. at reference bus 1, and limits the other buses from 0.9  
 to 1.1 p.u.. The transformer tap ratio  $\tau_i$  is imposed to 1. Each compensator  
 is with 0-80 MVar. The WTs and PVs are selected to represent the stochastic  
 DERs. The mappings of the DERs and buses are WT1:11, WT2:13, WT3:9,  
 PV1:2, PV2:7, and PV3:12. For simplicity, the capacity limit of each DER is  
 340 set to 18 MVA, and the related power factor is given as 0.90/WT and 0.95/PV.  
 The PDISCO rationally procures the DERs' portfolios and strategically trades  
 in individual markets with 24 times per stage. The purchase price  $\lambda_t^{DER}$  for  
 each DER's generation is assumed to be unique, as shown in Table I, which also  
 includes the sales price  $\lambda_t^{DSD}$ , and network operation cost  $C_t^{DS}$ . The DL multi-  
 345 plier  $\alpha_t$  is used to obtain the demand  $P_{tl}^{DSD}, Q_{tl}^{DSD}$  on the base value described  
 in [24].

For the market-operated 9-bus network: The PDISCO's network is assumed  
 to be interconnected at TL bus 4 with a power injection limit 100 MW. The  
 line capacity is set to 500 MW. Table II shows the capacity, reserve limit and  
 350 corresponding cost of each generator. The real-time regulation cost  $c_g^{UP}$  and  
 $c_g^{DN}$  can be regarded as the generation cost  $C_g^G$ . The TL multiplier  $\beta_t$  shown in  
 Table I is used to get the day-ahead bidding price  $\lambda_{td}^{TSD}$  relying on the base value  
 45 €/MW and consumption  $P_{td}^{TSD}$  according to the base in [25]. In addition,  
 the DL and TL load-shedding prices ( $\lambda_t^{SD}, \lambda_t^{ST}$ ) are considered as 200 times as  
 355  $\lambda_t^{DDA}$  and  $\lambda_{t\omega}^{DRT}$ , respectively.

Other DL and TL parameters can be found in [24][25]. All the price param-  
 eters mentioned above are estimated by the NordPool [26] prices.

Table 1: Essential Input Parameters for the PDISCO Tading in Markets

$t$ [Hour]	$\lambda_t^{DER}$ [€]	$\lambda_t^{DSD}$ [€]	$C_t^{DS}$ [€]	$\alpha_t$	$\beta_t$
1	1.0	12.0	1.6	4.0	0.3
2	1.9	22.8	3.0	7.6	0.3
3	2.0	24.0	3.2	8.0	0.3
4	2.5	30.0	4.0	10.0	0.3
5	3.5	42.0	5.6	14.0	0.8
6	4.3	51.0	6.8	17.0	0.8
7	5.0	60.0	8.0	20.0	0.9
8	5.5	66.0	8.8	22.0	0.9
9	5.8	69.0	9.2	23.0	1.0
10	8.0	96.0	12.8	32.0	1.7
11	9.0	108.0	14.4	36.0	1.8
12	9.6	115.2	15.4	38.4	1.8
13	9.0	108.0	14.4	36.0	1.7
14	5.5	66.0	8.8	22.0	1.0
15	4.5	54.0	7.2	18.0	1.0
16	4.8	57.6	7.7	19.2	1.0
17	5.5	66.0	8.8	22.0	1.0
18	8.3	99.0	13.2	33.0	1.5
19	8.5	102.0	13.6	34.0	1.6
20	9.0	108.0	14.4	36.0	1.7
21	9.5	114.0	15.2	38.0	1.8
22	3.5	42.0	5.6	14.0	0.8
23	2.1	25.2	3.4	8.4	0.5
24	1.5	18.0	2.4	6.0	0.4

Table 2: Input Parameters for Generators

No.	$\bar{P}_g^G$ [MW]	$C_g^G$ [€/MW]	$C_g^{UP}$ [€/MW]	$C_g^{DN}$ [€/MW]	$\bar{R}_g^{UP}$ [MW]	$\bar{R}_g^{DN}$ [MW]
1	1500	27	30	22	500	500
2	1200	29	32	24	300	300
3	1200	36	38	30	300	300

#### 4.2. PDISCO trading strategies

The PDISCO's strategic offers and bids make up continuous curves for trading in the two-stage market, as shown in Fig.2. As expected, in the day-ahead market, the PDISCO behaves as an active producer to provide production in hours 1-9, 14-17, and 22-24, with lower offering prices (around 28 €/MW). In the remaining hours, the PDISCO performs as an active consumer to acquire power at higher prices (about 32 €/MW or more). For illustrative purposes, in a scenario-based ( $\omega=1$ ) real-time market, sequences of the PDISCO's strategies are also presented in Fig.2. Lower prices (approx. 14 €/MW) are consistent with the power offers as in hours 1, 3, 8-9, 14-17, and 23-24. Power bids appear along with the higher prices (45-49 €/MW) at hours 2, 4-6, 10-13, 18-20, and 22. However, low prices (14 €/MW) are also conducted at hours 7 and 21 during the PDISCO's real-time bidding process. The reason for this special phenomenon is that the PDISCO's trading strategies are made across the two-stages markets, implying the PDISCO's arbitrage behavior, i.e. the PDISCO submits a strategic offer/bid to raise the system-wide upward reserve beyond the actual required and thus cleared in the day-ahead market. Subsequently, in the real-time market, when the PDISCO bids more but has not reached the artificially high system-wide upward reserve, the cleared price falls to a much lower point. Taking this price, the PDISCO can avoid certain comparatively high prices in real-time markets and thus gain more profit. Specifically, the PDISCO hands over a proper offer and a crucial bid at day-ahead hours 7 and 21, individually, to avoid the minus-profit at the corresponding real-time hours.

Coordinating the trading strategies, the PDISCO simultaneously makes rational acquisitions on the DER portfolios at the day-ahead stage. The details of the PDISCO's procurements are shown in Fig. 3. Large amounts of DERs are motivated in the peak periods, i.e., hours 10-13 and 18-21. Throughout the day, WTs render relatively high outputs, since the PVs' generation declined significantly during hours 1-4 and 23-24. Consequently, the PDISCO's profits by trading in markets are obtained and shown in Fig. 4. Scenario-based ( $\omega=1$ ) real-time profits are recorded for comparison with the day-ahead profits. Ob-

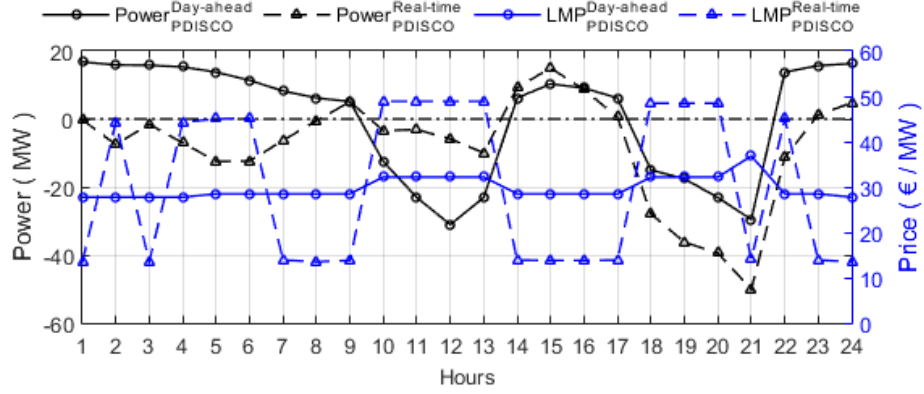


Figure 2: The PDISCO's strategic offers and bids in two-stage markets.

serve that, the day-ahead profit peaks occur consistently with the load peaks.

390 In contrast, the real-time profits have few volatilities, which are basically recognized as minus-profits to cover the deviations of DERs' generations, such as hours 2, 4-6, 13, 18-20, and 22. This leads to a 10% reduction in the PDISCO's total profit. Note that hours 7 and 21 are the PDISCO's bidding periods, while the related profits are retained at around non-negative values. These results  
 395 demonstrate the arbitrage schemes mentioned above are functional, i.e., a moderate arbitrage involved at hour 7, and a heavy arbitrage realized at hour 21.

#### 4.3. Impact on TL generators

The PDISCO's offers/bids can vary the markets' outcomes by interrelating with the TL generators' offers. For brevity, the cleared power quantities of generator 1 are taken as an illustrative example to address the PDISCO's impacts  
 400 on the markets, as shown in Fig. 5. The comparative situation is carried out on the TL network without the PDISCO's trading strategies and executed high real-time deviation ( $\pm 10\%$ ) on each DL demand. The generation offered and reserves committed (upward and downward) at the day-ahead stage are exhibited in Fig. 5(a), and the real-time up and down regulation power are outlined  
 405 in Fig. 5(b). Observe that, the PDISCO's involvement results in a dramatic



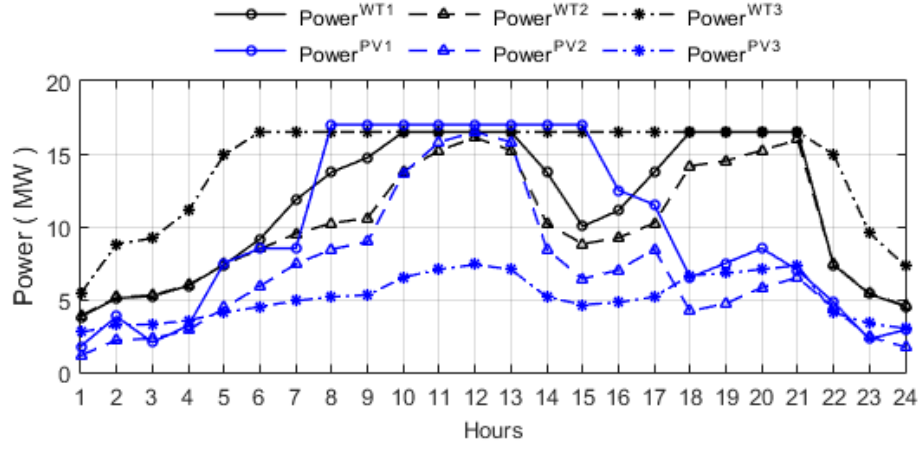


Figure 3: The PDISCO's power procurements from individual DERs.

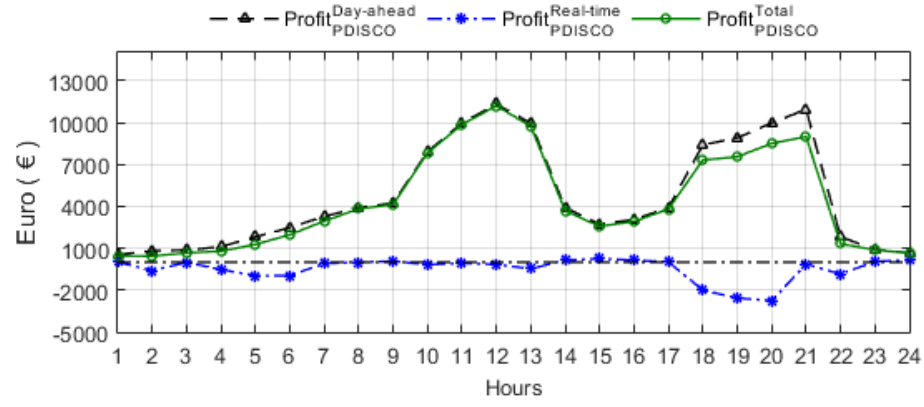


Figure 4: The PDISCO's day-ahead, real-time and total profits.

generation drop for generator 1 on day-ahead offers, in total 19% per day. In order to mitigate the unpredictable oscillations of the PDISCO's offers/bids, the notable power increment occurs in both reserve and regulation circumstances, i.e., 192% (71%) and 553% (56%) for daily upward (downward) reserve and up (down) regulation, respectively. The peaks of the upward reserve and up regulation capacity are coincidentally launched at hour 21, corresponding to the PDISCO's heavy arbitrage scheme indicated above as for Fig. 2 and 4. The effect of the PDISCO's moderate arbitrage at hour 7 is not explicitly expressed, since the upward reserve and up regulation power of the generator 1 are already

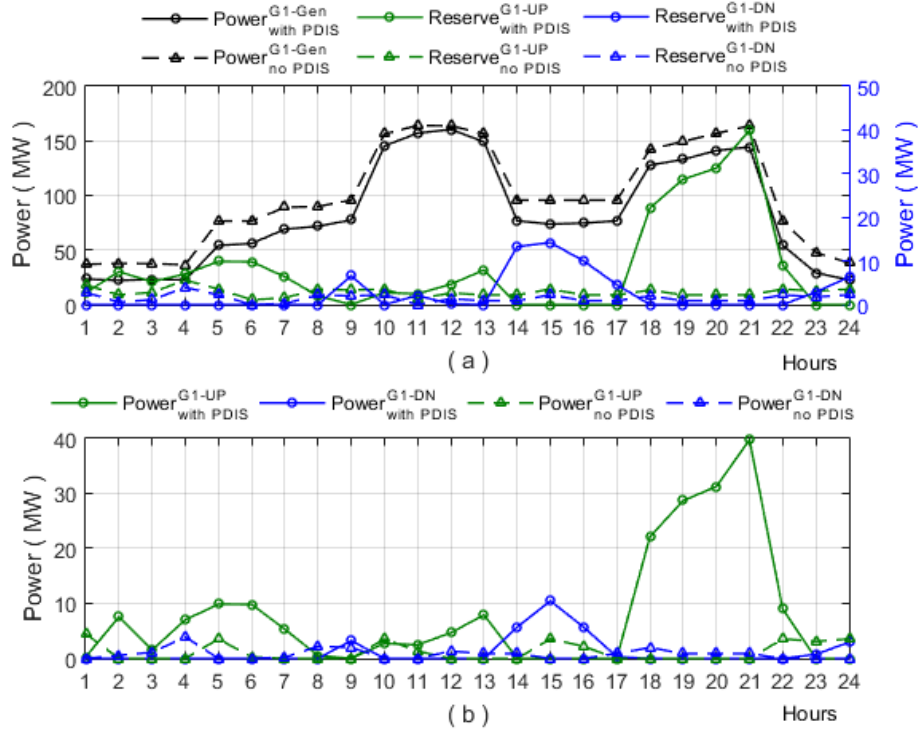


Figure 5: Cleared power quantities from markets for generator 1.

#### 4.4. Impact on TL markets

In addition, the comparison results also include the day-ahead social welfare and real-time operation cost of the markets, as shown in Fig. 6. The social welfare continuously declines by considering the PDISCO's participation, reducing by 12% daily. In contrast, the cost increases by 962% for the real-time market operation. In other words, the PDISCO's strategic trading negatively impacts the markets' objectives.

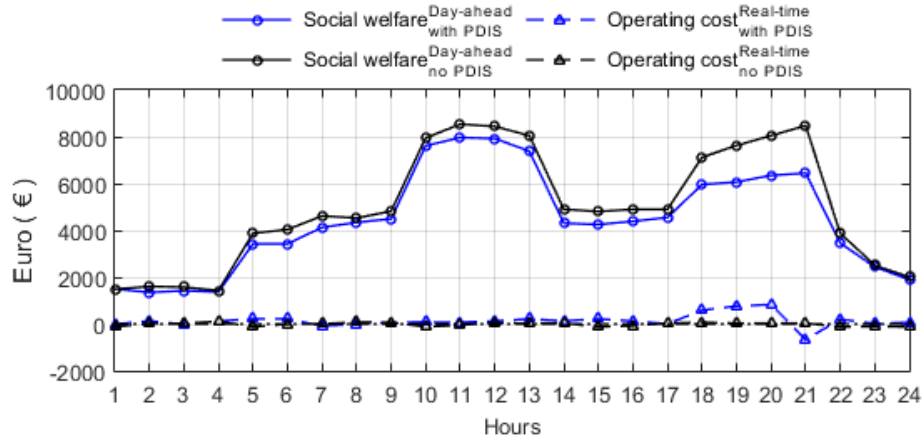


Figure 6: Day-ahead social welfare and real-time operation cost of markets.

Concerning the PDISCO's trading strategies essentially depend on the DERs' capacities, we further focus on the DERs' impacts on the PDISCO's decisions by resetting the DERs' availability in additional cases. Thus, the discussions above are listed as Case 1. Case 2 increases the individual DERs' capacities to two-times higher than Case 1, while these are reduced to half in Case 3. The results of PDISCO's profits and markets' objectives are summarized in Table III. The obtained social welfare is almost the same in Cases 1 and 3, but falls by 13% for Case 2. This means higher penetration of DERs yields more offers from the PDISCO and lower social welfare in the day-ahead market. The operation cost of the real-time market increases significantly with the DERs' capacity growth in Cases 1-3. Observe that the higher DERs can not ensure a

435 higher profit for the PDISCO, while lower DERs definitely bring lower profit, and even render minus-profit (in Case 3). To improve the profitability for the PDISCO, appropriate portfolio procurement is the best option.

Table 3: Achievements of the PDISCO and Markets

No.	PDISCO profit [€]	Day-ahead market social welfare [€]	Real-time market cost ( $\omega=1$ ) [€]
Case 1	11334.8	10642.9	339.5
Case 2	10457.5	9300.8	1497.9
Case 3	-54138.2	10654.4	237.0

## 5. Conclusion

This paper presents a bilevel model for a PDISCO strategically trading within the TL two-stage markets, considering rational procurements of DERs' portfolios. Crossing the markets, the continuous offers and bids are optimized to maximize the PDISCO's profit, accompanying moderate and heavy arbitrages. Interacted with the PDISCO's strategies, the social welfare is maximized in the day-ahead market, and the operation cost minimization of the real-time market is achieved in each scenario  $\omega$ . The proposed model is translated to a single-level solvable MPEC with the primal-dual approach. The results of the case studies reveal that the PDISCO's strategic offers and bids are effective to maximize its profit by trading in the wholesale markets. Some relevant remarks are as follows.

450 1) The newly defined market player (PDISCO) and presented DL DER trading turn the traditionally passive distribution grid into a proactive fashion. The liberalization of a DL market is an essential part in the whole chain of trading electricity, transacting with the wholesale market and acquiring DERs.

2) The increasing electricity purchases from stochastic DERs can not definitely guarantee a higher profit for the PDISCO. However, the proposed methodology can assist the PDISCO to seek the appropriate portfolio procurements from DERs in differing production scenarios.

3) A PDISCO can be seen as a centralized renewable energy source (e.g., large-scale wind or PV farms), which can be considered to integrate in the wholesale markets. This may facilitate the DL purchases of DERs to interact with the TL markets in real life. Besides, the DL trading between DERs and the PDISCO can be extended to multiple mechanisms with various clearing procedures, such as ancillary market, reserve market, and flexibility market, which could enhance the energy efficiency and system reliability.

4) As the interface actor, the PDISCO's procurement with DL DERs and trading in the TL markets are exhibited as hierarchical structures, which can be formulated by a tri-level model. The corresponding modeling and solving approaches can be investigated in the future.

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